A test of first order scaling of the N_f=2 QCD phase transition

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We complete our analysis of N_f=2 QCD based on the lattice staggered fermion formulation. Using a series of Monte Carlo simulations at fixed $m_L L_s^{y_h}$ one is able to test the universality class with given critical exponent y_h. This strategy has been used to test the $O(4)$ universality class and it has been presented at the previous Lattice conferences. No agreement was found with simulations in the mass range $am_q = [0.01335, 0.15]$ using lattices with $L_s = 16$ up to 32 and $L_t = 4$. With the same strategy, we now investigate the possibility of a first order transition using a new set of Monte Carlo data corresponding to $y_h = 3$ in the same mass and volume range as the one used for $O(4)$. A substantial agreement is observed both in the specific heat scaling and in the scaling of the chiral condensate, while the chiral susceptibilities still presents visible deviation from scaling in the mass range explored.

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1. Motivation

The order of the chiral transition of two-flavor QCD is still debated. Its relevance is both phenomenological (heavy ion experiments) and theoretical: is the transition an order-disorder transition associated with a change of symmetry?

With massless quarks the chiral/deconfinement transition for N_f=2 is expected to be second order in the $O(4)$ universality class or first order. Two completely different scenarios correspond to those two possibilities. Since, contrary to the first order case, second order phase transitions are unstable against the explicit breaking of the underlying symmetry, in the second case one would have a crossover instead of a real phase transition for small but non-zero quark masses. That would imply the possibility of going continuously from a confined to a deconfined state of matter, in contrast with the idea of confinement being an absolute property of strongly interacting matter at zero temperature and of deconfinement being an order-disorder transition associated to a change of symmetry [1]. A second consequence would be the presence of a crossover line, at finite mass, in the region of high temperature and small baryon chemical potential μ_B of the QCD phase diagram, thus implying a critical point [2] to connect with the first order line which is supposed to exist at low temperatures and large chemical potentials. No such point is expected to exist if the transition at $\mu_B = 0$ is first order. No critical point has been found up to now in experiments with heavy ions, but the question is still open [3, 4, 5].

2. The method

To establish the order of a phase transition using Monte Carlo simulations is not an easy task. Since all quantities are analytical in a finite volume, one must study the finite size scaling (FSS) to look for developing singularities. Here the FSS analysis is made intricate by the fact that, since simulations at zero quark mass are not feasible, one has to consider both the finite spatial size L_s and the finite quark mass $m_L = am_q$. The FSS ansatz for the singular part of the free energy density F_s around the chiral critical point is:

$$F_s(\tau, m_L, L_s) = L_s^{-d} F_s \left(\tau L_s^{1/\nu}, m_L L_s^{y_h} \right), \tag{2.1}$$

where τ is the reduced temperature $\tau = (1 - T/T_c)$, ν and y_h are critical indexes. From the above equation we can derive the FSS of the specific heat:

$$C_V - C_0 \simeq L_s^{\alpha/\nu} \phi_c \left(\tau L_s^{1/\nu}, m_L L_s^{y_h} \right), \tag{2.2}$$

and of the susceptibility χ_m of the chiral condensate:

$$\chi_m - \chi_0 \simeq L_s^{\gamma/\nu} \phi_\chi \left(\tau L_s^{1/\nu}, m_L L_s^{y_h} \right). \tag{2.3}$$

In Ref. [6] we have approached the problem varying m_L and L_s in such a way that the scaling variable $m_L L_s^{y_h}$ was fixed, thus reducing the FSS Eqs. (2.2) and (2.3), to one variable. Since the critical exponent y_h must be fixed, one can only test a given universality class, which in Ref. [6] was chosen to be $O(4)$. In that case no sign of scaling with the $O(4)$ critical exponents was found. In the present work we repeat the analysis fixing $y_h = 3$ as expected for a first order transition.
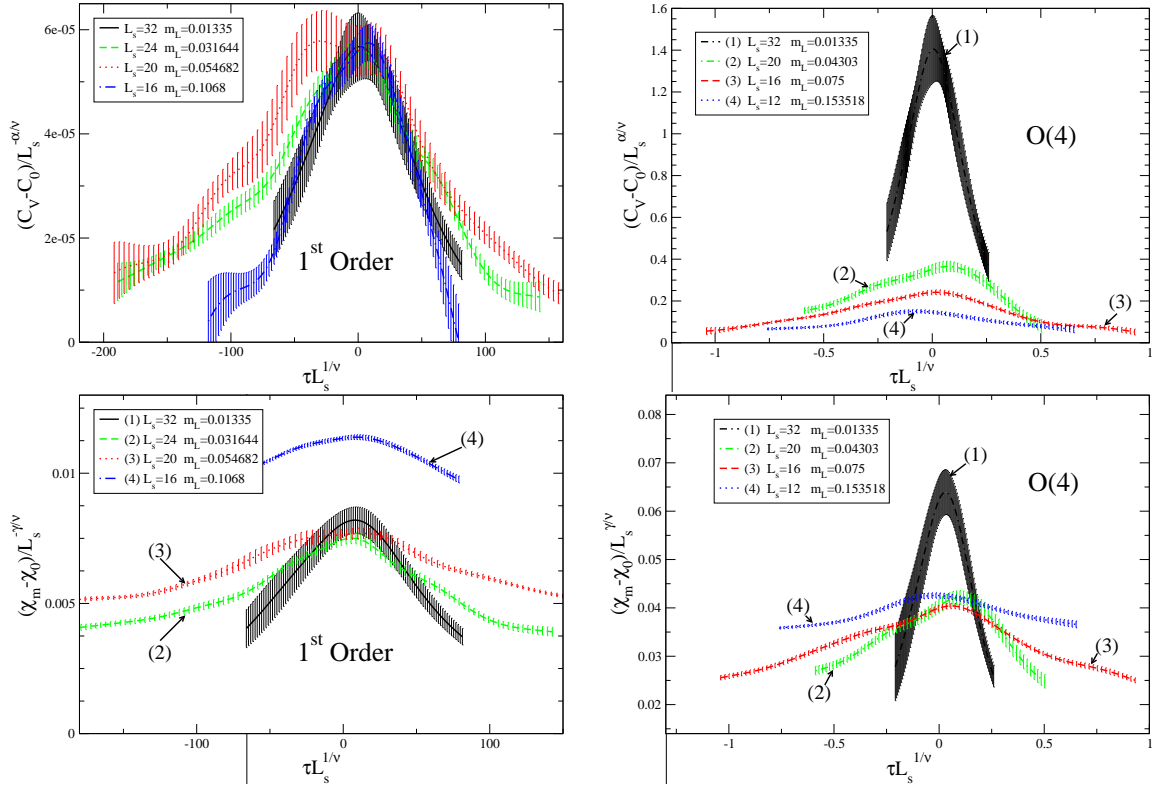
Figure 1: Comparison of first order and $O(4)$ scaling of C_V and χ_m.

3. Simulation details

As in Ref. [6], the lattice discretization used is the standard staggered action. The algorithm used in the new simulations for the test of first order scaling, is the RHMC. All lattices have $L_t = 4$. New Monte Carlo data have been generated to match the $L_s = 32$, $m_L = 0.01335$ lattice used in Ref. [6], thus fixing the parameter $m_L L_s^3$. New datasets for lattices with spatial extension $L_s = 16, 20, 24$ have been produced. For each of them ten different values of β spanning the entire critical region were simulated with a total statistics of about 90k thermalized trajectories collected for each of the lattices. The data collected is analized using a multi-histogram reweighting to extrapolate the observables to intermediate couplings. The backgrounds C_0 and χ_0 appearing in Eqs. (2.2) and (2.3) are determined as in Ref. [6], i.e. C_0 from a linear fit of the tails of the measured plaquette susceptibilities, and χ_0 is fixed by a fit of the maxima of the chiral susceptibility in the critical region.

4. Test of scaling

In Fig. 1 the scaling expected for a first order transition for the specific heat C_V and of the chiral susceptibility χ_m is shown. The picture also includes the corrisponding figures for $O(4)$ from Ref. [6]. For the first order case a reasonable scaling is observed for the specific heat C_V. As remarked in Ref. [6], C_V is independent of any prejudice on the symmetry and on the order parameter. Violations of the scaling Eq. (2.3) are observed for χ_m at larger values of the masses. In
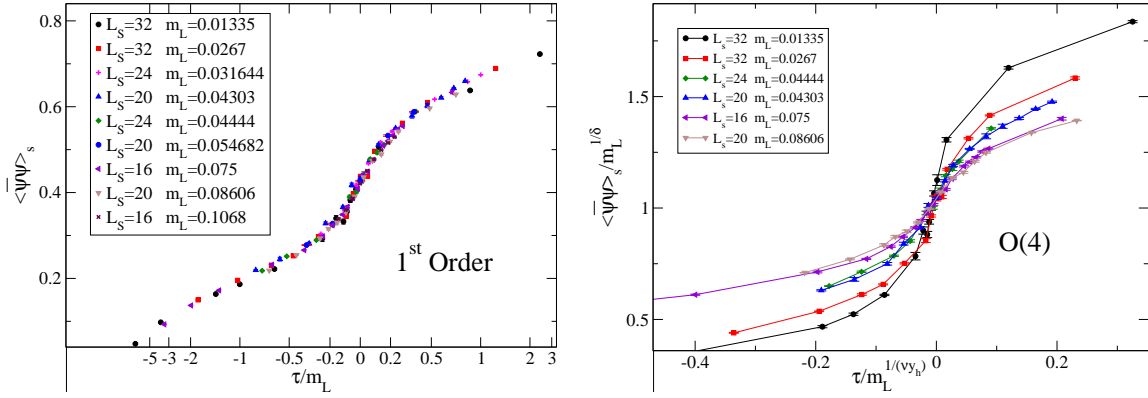
Figure 2: Comparison of first order and $O(4)$ scaling of the subtracted chiral condensate.

fact Eq. (2.3) is expected to be valid for the susceptibility of the order parameter. At large masses chiral symmetry is badly broken and $\langle \bar{\psi}\psi \rangle$ is possibly not a good order parameter. In our data, both of the present paper and of Ref. [6], Eq. (2.3) seems to be violated for $m_L > 0.05$.

The scaling of the chiral condensate (magnetic equation of state) can also be checked. The comparison between first order and second order $O(4)$ is shown in Fig. 2. The first order scaling is very good. As in Ref. [6], the scaling of the chiral condensate is obtained after a subtraction proportional to m_L as $m_L \to 0$. No sign of scaling is observed for the second order case.

5. Conclusions

The direct test of first order scaling shows a good scaling for the specific heat and for the chiral susceptibility at masses $m_L < 0.05$. At larger masses scaling of the chiral susceptibility is broken, presumably because of strong breaking of the chiral symmetry. By comparison with the similar test for $O(4)$, the first order scaling is clearly preferred over the second order. We can say that $O(4)$ is inconsistent with the lattice data while a first order transition describes well the observations. Possible effects due to the discretization must be taken into account. We believe that ultraviolect effects should be irrelevant with respect to the large volume behavior. However the use of finer lattices and new simulation algorithms to approach the chiral limit will possibly clarify this issue.

Acknowledgments

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